Eranism: a Need-Based Aid Economy

Part 1: Basic Mechanism

Arnold J. Bomans*

June 29, 2021 draft†

Abstract

The Eranist economy derives allocation weight from individual labour time and constrains the common labour time, possibly also the amount of goods. It should contribute to a moneyless, mutualist, and purposeful society.

*abomans@hotmail.com

†The margins indicate possible improvements. Syntactical, linguistic, and stylistic errors are to be corrected.

Contents

1 Introduction

The word 'Eranism' is derived from the classic greek 'eranos' which means assistance association [11] and no-host dinner [8]. Eranism is a generalization of this: an economy where one assists only if needed and expects to be helped when oneself is in need, but not necessarily by the person one has helped before. That is, the economy is not based on reciprocation and the resulting obligations but on mutualism. (Obligations such as of contract remain.) Consequently, there is no need for money, the tradeable materialization of obligations.

The eranist economy allows to avoid two root causes of environmental problems and unfair inequality:

1. competition, and

2. debt, in particular, money.

Part 2 elaborates this motivation. Part 3 gives rationing algorithms which discourage to misreport one's needs. Part 4 is a database design. This is Part 1, the basic mechanism of Eranism.

The eranist economy has three components.

Time pool A time pool containing labour times of employees ('assistants') which are entered by the employer (the 'client'). These times can not be traded so the system differs from time banks. The collective labour time is bounded from above. The upper bound deters people from doing unnecessary work and thus keep time for necessary work. "I am not going to write more hours than you worked just because you are my friend: I soon might be needing time to have some other job done. That holds for you, too."

Effort A personal "effort" which equals labour time divided by one's potential labour time. This fraction serves as a weight when obtaining goods or services. It should be an incentive to work, but not lead to a race for increasing one's weights, for that would result in unnecessary work. "See what I can do this week because I do not want to rely on what is left over."

Limits The determination of the amount of available goods (the limits) for the collective and for the individual. If one claims that more goods

> need to be produced than one needs in reality, then the upper bound to the collective working time is raised, so this discourages, to a certain extent, asking too much. “Hm, perhaps I should not ask for one more bread-de-luxe because I do not like extra weeding this week just to raise my effort.” Once the goods are there, no profit is gained from reporting more than one really needs. (It is strategy-proof, see Part 3.) Roughly speaking, and for the case of equal effort: the more one asks, the later one is served, though one also shares in what is left over from those served early-on. This a metaphor because no real queue is needed.

In summary, reciprocation would be replaced with an incentive to members of a community to help.

These component have some roots in history. For instance, many communists and socialists envisaged a non-monetary society [6, p.33ff]. Mutualism from an economic point of view is present in the use of labour notes [5] and in distributism [2]. However, none of these economies employ strategy-proof time pooling or allocation.

As to contemporary currents: literature about strategy-proof production economies has been reported to be scarce [1, p.799]. One time-based approach to production economies is to choose individual, equal labour times to produce a public good [9]. Another is to consider leisure time as a good to be distributed [10]. Goods, as opposed to labour times, are often allocated without use of money (see Part 3 for details.) Impossibility results for production economies are, for example, in [4]. For attempts to list other non-monetary economies and rationing procedures, see Wikipedia. All in all, Eranism seems to be a novel approach to allocate goods (or services) and to arrange labour without money in a way that is strategy-proof and fair.

Finally, as to the scope: in principle, Eranism (as an economy) does not deal with property and decision making, but these delicate themes can not be ignored altogether. For, Eranism foresees that only certain administrators can enter the estimated amounts of available or needed resources. (The database design in Part 4 caters for this.) Estimating these amounts risks to be a time-consuming and specialized task, which therefore needs to be carried out by experts, who in turn work for a representative of the group. However, this raises the problem of representation: do all members of the group sufficiently trust the representative (and the expert) to precommit themselves to using portions of the estimated amounts? The problem of representation is also present below: every group has two designated persons who, in certain cases, enter labour time on behalf of the group. Fair arbitration of conflicts is another issue which is not dealt with by the eranist economy. Needless to say that at this stage, no path is sketched along which parts of society would transition to an eranist economy.

2 Mechanism

The details of the mechanism are as follows.

2.1 People and Time

Let P be as set of people. For $\epsilon > 0$ a small time step (like a minute) let $\mathcal{T} := \{\epsilon, 2\epsilon, \ldots\}$ be *time*. An *assistant* h from P starts to work at time τ during a *personal service time* $W_{h,\tau} > 0$ having the property that the *labour intervals* $I_{h,\tau} := [\tau, \tau + W_{h,\tau}]$ are disjoint for different τ. Let Q be a subset of P. Let $\mathcal{U}$ be a set of time units $u > 0$. Let $T = [t, t+u]$ be a *time interval* starting at time t and of length u, that is, $|T| = u$. Define

$$U_Q := \{(q,\tau) \in Q \times T \mid \ (q,\tau) \in \mathrm{dom}(W) \ \& \ \tau + W_{q,\tau} \in T\}$$

as the parameters of the intervals $I_{q,\tau}$ which are a subset of T. Let $\Theta := \{1, 2, \ldots\}$ be *entry moments*. Let λ be an injection from U_P to Θ with the property that $\lambda(p,\sigma) > \lambda(q,\tau)$ if and only if $\sigma > \tau$ for all persons p, q and for all times σ, τ. That is, ordering by entry moment respects the ordering by time. For each T, subset Q can have a *time budget* B_Q where $0 < B_Q \leq u$. If R is a subset of P which has a bound and which is disjoint with Q, then require $B_{Q \cup R} = B_Q + B_R$.

Definition 1 (Group Labour Time) *Define the set V_Q and* group labour time *L_Q of Q as follows. Let $\Xi_0 := \emptyset$ and $H_0 := 0$ as well as $N := |U_Q|$. For $i := 1, \ldots, N$ let (h_i, τ_i) be successive pairs from U_Q ordered by $\lambda(h_i, \tau_i)$. Define $X := H_{i-1} + |I_{h_i,\tau_i}|$ and the following two variables:*

$$H_i := \begin{cases} X & \text{if } X \leq B_Q \\ H_{i-1} & \text{otherwise} \end{cases} \quad \text{and} \quad \Xi_i := \begin{cases} \Xi_{i-1} \cup \{(h_i,\tau_i)\} & \text{if } X \leq B_Q \\ \Xi_{i-1} & \text{otherwise} \end{cases}$$

Finally, let $V_Q := \Xi_N$ and $L_Q := H_n$.

The N, H_i, X, and Ξ_ℓ are temporary variables. Define the *window* $\mathcal{W}_Q := \{I_{h,\tau} \mid (h,\tau) \in V_Q\}$. So the window is the largest set of labour intervals of Q fitting T and starting at the oldest interval and skipping intervals which exceed the total time B_Q in the order of the entry moments. Notice that a labour time which at one moment makes the bound B_Q to be exceeded, may be counted later-on or for a longer time unit. Let $\Delta_Q := L_Q - B_Q$ be the *remaining labour time.* So $L_Q = \sum_{(h,\tau)\in V_Q} |I_{h,\tau}|$ and $\Delta_Q \geq 0$. Introduce $S_h := \sum_{\tau\in T:(h,\tau)\in V_Q} |I_{h,\tau}|$ as the *total labour time* of h in Q. So $L_Q = \sum_{h\in Q} S_h$.

2.2 Groups

A *sample group* A is a relatively small set of two or more persons who can assist and whose professions in the set are as divers as possible. Each A has a time budget B_A.

A *client* $n = \nu(h,\tau)$ from P is someone who enters the labour time of h and he or she is the only one who can do so. The *registered labour time* is $\widetilde{W}_{h,\tau}$. It may be larger than the true labour time $W_{h,t}$ if h reports too much to increase his or her weight (the principal agent problem) or if n reckons that h will reciprocate the overreporting. It may also be less if n wants to avoid too high a weight of h or if h reports less to n so as to create more rome for help later-on, either for h or for n. Let $\widetilde{\mathcal{W}}_A$ be $\mathcal{W}_A$ where each $W_{h,\tau}$ has been replaced with $\widetilde{W}_{h,\tau}$. Similarly for $\tilde{L}_A$ and so on. To make a labour time obey the restriction imposed by T and B_Q, the time may be split in multiple labour times or the reported labour time may be reduced until it fits. Proposition 1, p.8 will elaborate on this.

If work is to be proposed to assistants (for example, using the uniform rule for 'bads') then this is done by a *client group*, which is a group of clients. As this requires the clients to convene very often, it is not considered further now. People in need for goods are collected in the same way in *demander pools*, which bear no relation with sample groups. A *community* is a group of people who profit from the work of an assistant. For now, it coincides with the sample group. A *communal assistant* is an assistant who works for a community. Two or more clients should be *representatives* of a community. They enter labour time on behalf of the community, typically of a communal assistant, but they can also register the labour time of a painter who paints his or her own house. Somebody who outsources work temporarily is a representative as well.

2.3 Partition of Sample Groups

Let $\mathcal{A}_1 \leq \mathcal{A}_2 \leq \cdots$ be partitions of P into sample groups, where $\mathcal{A}_1$ is finer than $\mathcal{A}_2$ and so on. (The top partition need not be the whole of P.) The sample group $A_{p,q}$ of persons p and q is the sample group from $\mathcal{A}_1$ containing p and q if one exists, otherwise from $\mathcal{A}_2$ if one exists, etcetera, and otherwise there is no such sample group. The group of p is $A_{p,p}$. This allows a profession to often be used in an encompassing sample group. As of now, labour time is entered by a client of the sample group.

Now suppose that a client n needs work from helper h and no sample group $A_{n,h}$ exists. Without further measures, help by h to n would only reduce the time budget of $A_{h,h}$ and therefore would not be advantageous to $A_{h,h}$ but only to h if h wants to increase his or her weight despite the budget decrease. To let it be profitable, define a *sample group pool* as the collection of sample groups $A_{m,g}$ where $m \notin A_{h,h}$ needs help from $g \in A_{h,h}$ and no $A_{m,g}$ exists. A representative of $A_{h,h}$ would let h wait for a pool of at least two sample groups to form, as long as is opportune for $A_{h,h}$ to have the following work out. If such a pool is formed, then for a long enough time unit u and for each Q in the pool, let L'_Q be the amount of time which Q spent to other groups than Q in the same way as $A_{n,n}$ did. That is, in Definition 1, p.5 sum over (h_i, τ_i) where h_i in Q and $n_i := \nu h_i, \tau_i$ not in Q while no A_{n_i,h_i} exists. The help by h is granted to the Q having the greatest L'_Q/L_Q. So, $A_{n,n}$ may be helped or some other Q. So, the "altruism" of the winning Q is rewarded. Such a reward may also once be given to $A_{h,h}$ so this mechanism is an incentive for $A_{h,h}$ to give "altruistic" aid.

2.4 Weights and Priorities

For all h in P let there be a *personal time budget* $B_{\{h\}}$. So $\tilde{L}_{\{h\}} \leq B_{\{h\}}$. Goods are allocated using weighted gains; when indivisible, a slight modification makes it a sequential allotment. (See part 3 of this series.) The *weight* of the person h is $w_h := L_{\{h\}}/B_{\{h\}}$, that is, the labour time as a fraction of the maximum labour time, for various time units u. Note that the weights hold for every good (or labour, if assigned) but that they may fade or be surpassed by weights of others as time goes by. Both the weights and the need to produce goods should be incentives to work. When allocating goods, communal assistants get *priority* over assistants who work for a client who does not represent a community. This priority is to be determined by that community. (Weights could also be derived from the demander pool, but this would complicate matters.)

2.5 Properties

The allocation of goods is based on 1. reported needs, which are private during the allocation but which can be guessed on the basis of previous allocations; and 2. weights (the duration of actual work as percentage of the potential duration) which are known to a certain extent because one works together. Still, for weighted gains allocation and its variation for indivisible goods (which is a case of sequential allotment) somebody needs rather complete information to judge whether increasing the labour time of an assistant is a threat for the one's allocation, namely: the complete processing order of people, all weights, and for those processed before the person, all reported needs. Therefore, it is realistic to assume that is no such information is available.

Proposition 1 *The following only holds for the next instance of assistance so further instances are ignored. Suppose participants do not know whether increasing the labour time of one person is a threat to the allocation of a specific other person more than to any other person. Yet, the time budget and the total labour time so-far are publicly known.*

For a client, there is no incentive to record less than the true labour time if this is needed to enter labour time at all. Underreporting labour time in other cases does offer some potential advantage to the client, unless a relation or a reputation is to be established, in which case it is a disadvantage. If the order of Neither does a client have an incentive to record superfluous labour time, that is, more time than an assistant worked in reality or time for unnecessary work, provided this is not to establish a relation or reputation.

For an assistant, overreporting labour time to the client may or may not be profitable.

The proof is in the appendix.

Underrecording in case 2, p.13 is the problem of safe exchange [7] and strategic defaulting of time "payments." The problem that h may overreport to n, who enters the labour time of h (the principal-agent problem) is only partially solved. Sanctions on n for underreporting are not considered as communities should intervene the least possible.

2.6 Estimating the Time Budget

For the period T and the group A let labour times be estimated as follows. Let i be an identifier of the client, the kind of work, the prospective assistant, and so on. For $i = 1, \ldots, N$ let Z_i be expected labour times (in particular, for production of goods which are consumed at frequency near u) or reservations for unexpected labour. These times may be short to express, or to impose, scarcity of resources. Let $B_A := \sum_{i=1}^{N} Z_i$ be the time budget. Let $\nu(i)$ be the client who ordered the work. The Z_i is an estimate of

$$\sum_{(h,\tau)\in A\times T:\nu(h,\tau)=\nu(i)} W_{h,\tau}$$

which is the time assistants h have worked on job i during $W_{h,\tau}$ after a start at τ.

Proposition 2 *Consider some work (production or service) which can be carried out only by an assistant from A. Then everybody has an incentive to require the amount of such work he or she truly needs.*

Proof Suppose person n asks more work than needed. First, this decreases the weight w_n relative to other weights. Second, $L_A \leq B_A$, that is, $\sum_{h\in A} \tilde{S}_h \leq \sum_{i=1}^{N} Z_i$ so if some h refuses to carry out the unnecessary work, then n may have to work a longer $\tilde{S}_n$ than when n had not asked more than needed. ■

Once the goods have been produced, their allotment also encourages to report the true needs for goods because rationing along a fixed path or sequentially are strategy-proof. Antipication of this may also discourage overproduction, but not too high a service time.

The encouragement for truthful reporting is rather indirect and therefore, weak. So, the allocation of goods itself must be strategy-proof, for instance, using fixed-path or sequential allotment, as set out in Part 3.

3 Conclusion and further Research

Eranism discourages 1. asking more goods or labour than needed; 2. reporting more labour time than was spent in reality; and 3. underreporting labour time in order to ensure reciprocation, a good reputation, or to avoid sanctions.

Further research would be 1. trying out Eranism, first as a game, then in a small setting, 2. studying other types of allotment (such as bipartite rationing for service offers and lexicographic ordering) and 3. replacing the hierarchy with a semi-lattice. Finally, the last paragraph of the introduction mentioned issues which are not dealt with by the eranist economy. Settling these issues requires study of the scientific literature about collective decision making and the so-called tragedy of the commons.

A Proof of Proposition 1

Consider a client n and assistant h and time t so that $n = \nu(h, t)$. Let $A := A_{n,h}$. Let time $t' > t$ and an assistant h' in P constitute an actual or potential new instance of aid. Consider $u \geq t' - t + W_{h',t'}$ where $W_{h',t'}$ is the actual or expected labour time. So $\tilde{I}_{h',t'}$ is or will be contained in $T = [t, t+u]$. Define $\widehat{\mathcal{W}} := \{I_{h,t}\} \cup \widetilde{\mathcal{W}}_A \setminus \{\tilde{I}_{h,t}\}$ which is a window where at least $W_{h,t}$ is true but with no check on the bound B_A. To apply the bound B_A, choose u so that $I_{h,t}$ is contained in T. Let $\widehat{\mathcal{W}}' := \widehat{\mathcal{W}} \cup \{\tilde{I}_{h',t'}\}$ and $\widetilde{\mathcal{W}}' := \widetilde{\mathcal{W}}_A \cup \{\tilde{I}_{h',t'}\}$ so $\tilde{I}_{h',t'}$ is or will be the only new interval. These too are candidate windows to which B_A is to be applied.

Introduce some abbreviations: $W := W_{h,t}$ and $\widetilde{W} := \widetilde{W}_{h,t}$ and $W' := W_{h',t'}$ and $\widetilde{W}' := \widetilde{W}_{h',t'}$ and $B := B_A$ and so on. A third or further instance of aid is ignored so if h expects $\widetilde{W}' < W'$ then h may refuse work on the basis of a reduced weight alone and not because of some strategy involving a third aid instance.

Use the analogous definitions: $\hat{L} = \tilde{L} + W - \widetilde{W}$ and $\hat{L}' = \hat{L} + \widetilde{W}'$ and $\tilde{L}' = \tilde{L} + \widetilde{W}'$ and $\hat{\Delta} = B - \hat{L}$ and $\tilde{\Delta} = B - \tilde{L}$ and $\hat{\Delta}' = B - \hat{L}' = \hat{\Delta} - \widetilde{W}'$ and $\tilde{\Delta}' = B - \tilde{L}' = \tilde{\Delta} - \widetilde{W}'$. One has $\tilde{\Delta} > 0$, that is, counting $\widetilde{W}$ is possible, for otherwise there is nothing to talk about.

Define a *utility* of n as a tuple $U = (U_1, U_2, U_3)$ which is a function of $\widetilde{W}$ and $\widetilde{W}'$ and which has the following meaning: U_1 is the utility of help by h'; U_2 the utility of h having no weight; and U_3 the utility of h' having no weight. Let $U > V$ denote that the overall utility of U is greater than that of V.

Lemma 1 *Consider overreporting the labour of h: $\widetilde{W} > W$. If h' does not help n then this is not profitable to n.*

Proof Let y be the utility of the present event that $\widetilde{W} > W$. For comparison, let $x^{\pm}$ be the utility of the event $\widetilde{W} = W$, let x^{+} be $x^{\pm}$ in case n

is helped by h', and let x^- be $x^\pm$ in the event that n is not helped by h'. First of all, $x_1^- = y_1$ (no help by h') and $x_3^- = y_3$ (unchanged weight of h') but $x_2^- \geq y_2$ (true entry of the labour time of h yields a lower weight of h.) So $x^- \geq y$. Comparing x^+ to x^- depends on which inequality adds more to the total utility: $x_1^+ > x_1^-$ (help by h' or not) or $x_3^+ \leq x_3^-$ (the weight of h' is increased or not.) If n would have expected $x^+ \leq x^-$ then n would have chosen x^- but $x^- \geq y$. Would n have expected $x^+ > x^-$ instead, then $x^- \geq y$ implies $x^+ > y$ so in both cases, there is no utility for n to misreport. ■

Now for Proposition 1, p.8 proper. Simplify proof

Proof First, assume $n' := \nu(h', t')$ is in A. There are several cases.

1. $\hat{\Delta} > \tilde{\Delta}$, that is, $\widetilde{W} > W$: overreporting or the work was not needed. Note that $\hat{\Delta}' > \tilde{\Delta}'$. As relations and reputations are excluded, the analysis is restricted to the following. The overreporting may increase the weight of h. This disadvantage for n is not outweighed by any advantage, as the following shows.
   (a) $\hat{\Delta}' \leq 0$, that is, counting $\widetilde{W}'$ is impossible if W is reported. As $\hat{\Delta}' > \tilde{\Delta}'$, recording $\widetilde{W}$ does not change this, so in both cases, the prospects are the same.
   (b) $\hat{\Delta}' > 0$, that is, recording W would have allowed counting $\widetilde{W}'$.
      i. $\tilde{\Delta}' \leq 0$, that is, recording $\widetilde{W}$ did not allow counting of $\widetilde{W}'$. As to the perspective of assistant h: if $h' = h$ then overreporting by h made counting the next job impossible; if $n' = h$ then overreporting by h made help by h' impossible. Both cases were no use. Otherwise ($n' \neq h \neq h'$) overreporting to n actually is profitable for h. So, h will overreport if the last event has greater expected profit than the previous two cases.
      Now as to the perspective of the client n.
         A. $n' = n$ & $h' \in A$: no weight will be added to h' and h' knows this, so h' will not help n. As the lemma shows, this is not profitable for n. (If $h' = h$ and the impossibility of counting $\widetilde{W}'$ is worse than the increased profit of h then the relation between n and h' may be harmed, but that is not taken into account.)

B. $n' = n$ & $h' \notin A$: Item 1(b)iA, p.11 applies to $A = A_{n,h'}$ if it exists, otherwise (if there is no common sample group) then $\widetilde{W}$ does not influence whether h' would have helped or not. If $h' = h$ then the additional effect is as for Item 1(b)iA, p.11.

C. $n' \neq n$ & $h' = n$: the weight of assistant n is not increased. (If $n' = h$ then h may agree to help though $\widetilde{W}'$ will not be counted: this would be a reciprocation for overreporting W but this argument would be for a relation or reputation, which is excluded.) Note that h' is in A.

D. $n' \neq n$ & $h' \neq n$: not applicable as n is not involved. (For $n' = h$ see Item 1(b)iC, p.12 and for $h' = h$ see Item 1(b)iA, p.11.)

ii. $\tilde{\Delta}' > 0$, that is, overreporting allowed counting $\widetilde{W}'$. For assistant h this is clearly profitable. Now consider the perspective of client n. Only the cases 1(b)iA, p.11 and 1(b)iC, p.12 need to be treated.

A. $n' = n$ & $h' \in A$. First, the perspective of h: the increased weight is profitable or, if $h' = h$, does not make a difference. Now, the perspective of n. If h' did not help n (because h' was disappointed that $\widetilde{W}$ was not as large as it could have benen) then the lemma tells that overreporting was not profitable to n. Now, suppose that h' helps n (perhaps h' badly needed the weight, however disappointing its size is.) Let e be the utility of the event that $W = \widetilde{W}$ (truthful reporting) and let f be the utility of the present event. Then $e_1 = f_1$ (help in both cases) and $e_2 \geq f_2$ (the weight of h may increase) but $e_3 \leq f_3$ (the weight of h' may decrease.) As $\hat{\Delta}' > \tilde{\Delta}'$, overreporting lets more weight be distributed amongst h and h'. If $h \neq h'$ then n does not have enough information to decide which inequality contributes most to the utility, so n must assume that $e > f$, in other words, n gains nothing. If $h' = h$ then certainly $e > f$.

B. $n' \neq n$ & $h' = n$. As to the perspective of the client n: assistant n may gain a higher weight if that is worth the effort but less than would have been possible if W were reported truthfully, $W = W'$. So, this is only a disadvantage. As to the assistant h: if $n' = h$ then h would be favoured again by n, not because n overreports but because n helps h despite a reduced weight gain. The increased risk of not being helped is a disadvantage for h.

2. $0 < \hat{\Delta} < \tilde{\Delta}$, that is, $\widetilde{W} < W$: underreporting. This diminishes the weight of h, which is clearly no advantage for h. Client n does not have enough information to decide whether this prevents allocation of goods to n which otherwise would be for h; client n can only hope to earn weight if assistant at the next instance. Unless stated otherwise, the following is about the perspective of n. It shows that there is no disadvantage for n except when h retaliates or the reputation of n is damaged.

   (a) $\tilde{\Delta}' \leq 0$, that is, underreporting does not allow counting $\widetilde{W}'$. As $\tilde{\Delta}' > \hat{\Delta}'$, truthfully recording does not either. So, this does not make a difference.

   (b) $\tilde{\Delta}' > 0$, that is, underreporting allowed counting $\widetilde{W}'$.

      i. $\hat{\Delta}' \leq 0$, that is, entering W would not have allowed to count $\widetilde{W}'$. In all likelihood, h' would not have helped n'.

         A. $n' = n$ & $h' \in A$: let the utility of the present event be called f and let e be the utilituy of $W = \widetilde{W}$. Then $e_1 < f_1$ (aid to n is better) but n lacks information to decide whether $e_2 \leq f_2$ (a lower weight of h is good for n) and whether $e_3 \leq f_3$ (a higher weight of h' is bad for n.) So n must assume $e_2 = f_2$ and $e_3 = f_3$. Therefore, $e \leq f$. However, h may take revenge later-on.
         As to the perspective of h: if $h' = h$ then the underreporting of W is not compensated by the fact that this allowed entry of W' for h; although n can enter $\widetilde{W}' > W'$ and thus try to restore the relation, h can not reckon on this.

         B. $n' \neq n$ & $h' = n$: assistant n may gain weight, in contrast to the case that $\widetilde{W} = W$ if that is worth the effort and if enough labour time is counted for n'. If $n' = h$ then n risks retaliation.

ii. $\hat{\Delta}' > 0$, that is, truthfully reporting $\widetilde{W} = W$ also allowed counting a (mis)reported W', which is henceforth denoted by $\widehat{W}'$. If $\widehat{W}' \geq \widetilde{W}'$ then the extra room for reporting W' (as created by underreporting W) has not been used so this made no difference. Therefore, consider $\widehat{W}' < \widetilde{W}'$.

A. $n' = n$ & $h' \in A$: the increase of the weight of assistant h' is greater than would have been the case if W were reported truthfully. The sum of the weights of h and h' is not changed by underreporting and as the allocation can not be predicted by assumption, een not when $h = h'$, there is no further disadvantage for n.

B. $n' \neq n$ & $h' = n$: the increase of the weight of assistant n possibly is greater than would have been the case if W were reported truthfully. This is only an advantage. If $n' = h$ then the relation of n with h or the reputation of n will even be worse than after the underrecording by n.

3. $\hat{\Delta} \leq 0 < \tilde{\Delta}$, that is, underreporting was needed by n to count the labour time. Client n favours h because h gains some weight for the current time unit, possibly even surpassing the weight of n. Moreover, no time is left for n to help or to be helped.

(a) n obliges h to reciprocate. This in return forces n to register any remaining labour time later-on. (Registering for the future is not possible.) That is, the labour time will artificially be split and for a longer time unit, there is no difference with counting the original labour time. So, this is not an advantage for n and it may even be a disadvantage because after all, h may not do anything in return.

(b) n redeems an obligation towards h, which is good for the reputation of n.

The event that n' is not in A is only relevant if h' is in A, in which case the inter-group altruism should once lead to compensation of any increased weight of h'. ∎

References

[1] Barberà, S. (2002) *Strategyproof Social Choice*, in: Arrow, K.J.; Sen, A.; Suzumura, K. (eds.) *Handbook of Social Choice and Welfare*, vol.2, Gulf Professional Publishing.

[2] Cooney, D. W. (2020) *Distributism Basics. Foundational Principles*, USA: Bookpatch.

[3] Fadaei, S.; Bichler, M. (Jan. 2017) *Generalized assignment problem: Truthful mechanism design without money*, Operations Research Letters, vol.45, no.1, pp.72-76.

[4] Goswami, M.P.; Sen, A.; Yadav, S. (2015) *A Hurwicz type result in a model with public good production*, Social Choice and Welfare, vol.45, nr.4.

[5] Marshall, P. (1993) *Demanding the impossible. A history of anarchism.* London: Fontana (first printing 1992) On p.7: labour notes and mutualism conceived as reciprocity.

[6] Nelson, A. (2011) *Money versus Socialism*, pp.23-46 in: Nelson, A; Timmerman, F. (eds.) *Life without Money: Building Fair and Sustainable Economies.* London: Pluto Press.

[7] Sandholm, T.; Wang, X. (2002) *(Im)possibility of Safe Exchange Mechanism Design*, American Association for Artificial Intelligence, AAAI-02 Proceedings.

`https://www.aaai.org/Papers/AAAI/2002/AAAI02-052.pdf`

[8] Schneider, J.G. (1819) *Griechisch-Deutsches Wörterbuch beym Lesen der griechischen profanen Scribenten zu gebrauchen*, Leipzig: Hahn, 3d pr.

[9] Schummer, J.; Vohra, R.V. (2007) *Mechanism Design without Money.* In: Nisan, N.; Roughgarden, T.; Tardös, E.; Vazirani, V.V. (eds.) *Algorithmic Game Theory*, pp.243-264. Cambridge University Press. On pp.251-252: *Application to Public Good Cost Sharing.*

[10] Thomson, W. (dec. 2007) *Fair Allocation Rules*, Working Paper 539, Rochester Center for Economic Research.

http://idei.fr/sites/default/files/medias/doc/conf/ere/papers_2009/thomson-paper.pdf

[11] Van den Es, A.H.G.P. (1896) *Grieksch Woordenboek*, Groningen: J.B. Wolters, 5th pr.