Eranism

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Abstract

Brief sketch of changed design of Eranism, a moneyless economy.

1 Introduction

Since release, the model has been extended. Here is a quick and sketchy update. Employers now 'pay' labour hours from a 'basic income' to employees, but the employees can only use these hours for weight in the allocation. So, these hours are not a kind of money because input and output are in different dimensions.

2 Basics

Without further ado, here is the essence of Eranism. Consider only one group $N = \{1, \dots, n\}$ of people who have a variety of professions. The group has a fixed time budget B per time unit u for each member. (Whoever dislikes B can go to another group or create a group with the desired budget.) At the start a particular time period, every member reports preferences for goods of a certain type; they do so for various types of goods. Within the same time period, they create, in the role of candidate employers (or clients), vacancies for work of certain type. The employees (or assistants) carry out this work and some attestor (usually the client but never the employee himself or herself) records the labour time. At the end of the period, the labour times which have been recorded by a particular employer are summed: if the sum is less than the time budget of the employer (which is the same for each employer) then these labour times are finalised. If the sum exceeds the budget, the labour times are assigned according to the safety bounds bankruptcy formula (appendix) where the safety bounds are $\mu_1 := B/m$ for m the number of employees of the employer. Each work type has a maximum labour time per time unit: the time beyond which work is physically or mentally impossible (this would have to be determined by an independent organisation). The assigned labour times of an employee (for various employers) are added so that the maximum labour time is accounted: this yields an effort for the period. (If all work is equally hard, then this is just the sum of labour times.) Finally, the goods of each type are distributed by rationing along a fixed path (Heré Moulin) where the weights equal the efforts. This is the uniform rule where the efforts weigh to obtain the goods.

There is no point in 'paying' more labour time than has been worked in reality (e.g. to get an equal pseudo labour time or the extra goods thus obtained in return.) For, during the period, unexpected work may be needed but by then, the budget may be exhausted. This is why the group must contain a mix of professions.

Artificial rationing is possible. For multiple groups, things are more complicated but have already been programmed.

A Bankruptcy with Safety Bounds

A bank employs safety bounds if upon bankruptcy, investments below the safety bound are awarded to every customer. For claims above the safety bound, an amount is awarded that is a portion of the the safety bound and proportional to the investment.

An *endowment* E in $\mathbb{R}_{\geq 0}$ is to be distributed among n *claimants* $N := \{1, \ldots, n\}$ for some *number of claimants* n. Each claimant i has a *claim* c_i in $\mathbb{R}_{\geq 0}$ and a *safety bound* μ_i in $\mathbb{R}_{\geq 0}$; these entities will be further described shortly. To be determined is the *award* $S_i(c, \mu, E, n)$ in $\mathbb{R}_{\geq 0}$ that is granted to i. Its outcome is abbreviated to $x_i = x_i(c_i) := S_i(c, \mu, E, n)$. So $x_i(c_i)$ is an overloaded symbol for S_i when all variables except c_i are fixed.

For any function $f : N \to \mathbb{R}$, that is, for any $f = (f_1, \ldots, f_n)$, and for any subset M of N, let $f|_M := \{(j, f_j) \mid j \in M\}$ be the usual notation for the restriction of f to M. For any $g : M \to \mathbb{R}$ let $\mathbb{1}g := \sum_{j \in M} g_j$ where $\mathbb{1} := (1, 1, \ldots, 1)$ is a row of the appropriate dimension, $|M|$ in this case, and g is thought of as a column, so $\mathbb{1}g$ is an inner product. For n-tuples u and v let $u \leq v$ denote $u_i \leq v_i$ for all i in N.

The *bankruptcy problem* consists of determining x for the case $\mathbb{1}c \geq E$, which will always be assumed; the case $\mathbb{1}c = E$ is by convention. The condition of *efficiency* is imposed: $\mathbb{1}x = E$. source...

Consider the case $\mathbb{1}c < E$. The term 'claim' now also holds [3, p.252]. No x is defined but one can think of the bank gently closing down and $x := c$ so the claim c is returned. The remainder $E - \mathbb{1}c$ probably goes to the former bank owners.

The *bankruptcy problem with safe bounds* is the bankruptcy problem obeying the following conditions.

1. *safety bound condition*: for all i in N, from $c_i < \mu_i$ follows $x_i = c_i$. *Claims boundedness* [3, p.266]: $x < c$. (The term 'awards boundedness' would apply better.)
2. *Continuity*: x_i is a continuous function of c_i for all i in N. (It would be unfair or odd if an infinitesimal increase of the claim would yield a considerably larger award.)
3. *Monotonicity*: $c_i > c'_i$ implies $x_i > x'_i$ for all i in N. (Nobody wants to invest more but be awarded less.)

Continuity yields $x_i(\mu_i) = \mu_i$. From $c_i \geq \mu_i$ follows $x_i \geq \mu_i$ because x_i is monotonous, that is, claims above the safe bound yield an award also above the safe bound.

From the safety bounds conditions and efficiency follows $c \not< \mu$ so there is an i in N such that $c_i \geq \mu_i$. (Would $c < \mu$ then $\mathbb{1}c = \mathbb{1}x = E$, a contradiction.) It could happen that $c \geq \mu$. In that case, $E = \mathbb{1}x \geq \mathbb{1}\mu$. So, a necessary condition is $\mathbb{1}\mu \leq E$.

For example, most Dutch bank customers are guaranteed to receive their claims up to a standard safe bound $\mu_1 = 10^5$ euro from their bank when it goes broke – actually, from the central bank [2]. The terms 'guarantee' or 'lower bound' for μ_1 could be confusing because nobody who invested 1 euro will get 100,000 euro upon bankruptcy. Neither can μ_1 be called an upper bound to the award because some can get more. Suppose there are 10,000 customers and all have more than 100,000 euro on their bank account: $c \geq \mu$ and $\mathbb{1}\mu = 10^9$. So $E = \mathbb{1}x \geq \mathbb{1}\mu$. The bank does not own a billion euro and should never have promised these safety bounds: $E < \mathbb{1}\mu$. Fortunately, the central bank will come to the rescue. Of course, repeating this example for the central bank might not be as reassuring.

Judging from overviews by Thompson [3, 4] this bankruptcy problem has not been considered in the literature. Thompson remarks 'Imposing this bound [$c_{\max}$, on the claims] restricts somewhat the scope of the rule but it permits a very simple (piecewise linear) representation' [3, p.259, n.11] but that is a representation of the Talmud rule [1, p.284].

To derive a solution, consider first the (unsuccessful) trial solution $x_i = \min\{c_i, \mu_i\}$ for all i in N. Let there be a unique claimant j for whom $c_j < \mu_j$. Then $\mathbb{1}x = c_j + \sum_{i \in N \setminus \{j\}} \mu_j$ would hold, so $\mathbb{1}x < \mathbb{1}\mu \leq E$, a contradiction with efficiency. To amend this, try $x_i := \min\{c_i, \phi_i\}$ for some ϕ_i which is a function of μ and possibly other variables. Let $U := \{i \in N \mid \ c_i \geq \phi_i\}$ and $\overline{U} := N \setminus U$. Then $\mathbb{1}x = E = \sum_{i \in \overline{U}} c_i + \sum_{i \in U} \phi_i$ yields an implicit definition of ϕ because U depends on ϕ. Perhaps it can be solved by using some iteration, but a simpler approach is adopted here:

$$x_i := \begin{cases} \phi_i & \text{if } c_i \geq \mu_i \\ c_i & \text{otherwise} \end{cases}$$

Let $\phi_i = \alpha\mu_i + \beta c_i$ for some constants α and β so that, to a certain extent, ϕ_i is proportional to c_i (monotonicity) and ϕ_i contains a portion of the safety bound μ_i, but only when $c_i \geq \mu_i$.

Continuity yields $\mu_i = \phi_i = (\alpha + \beta)\mu_i$ so $\alpha + \beta = 1$. Let $V := \{i \in N \mid \ c_i \geq \mu_i\}$. From $\mathbb{1}x = E$ follows

$$\alpha = \frac{\mathbb{1}(c - \mu)}{\mathbb{1}(c - \mu)|_V}$$

after some calculus, which is to be combined with $\beta = 1 - \alpha$.

For example, $n = 4$, $E = 80$, $c = (10, 20, 30, 40)$, and $\mu = (11, 19, 21, 29)$. Then $V = \{2, 3, 4\}$ and $c - \mu = (-1, 1, 9, 11)$ so $\alpha = 20/21$ and $\beta = 1/21$. As a variation, let $\mu = (20, 20, 20, 20)$. Then $V = \{3, 4\}$ and $c - \mu = (-10, 0, 10, 20)$ so $\alpha = 20/30$.

References

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[2] Nederlandse Centrale Bank, https://www.dnb.nl/en/reliable-financial-sector/dutch-deposit-guarantee/

[3] Thompson, W. (2003) *Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: a survey*, Mathematical Social Sciences, vol.45, pp.249–297.

[4] Thompson, W. (2015) *Axiomatic and game-theoretic analysis of bankruptcy and taxation problems: an update*. Mathematical Social Sciences, vol.74, pp.41–59. http://dx.doi.org/10.1016/j.mathsocsci.2014.09.002 Also Working Paper No. 578, Rochester Center for Economic Research, https://rcer.econ.rochester.edu/RCERPAPERS/rcer_578.pdf